THINKING MATHEMATICALLY: A GUIDE FOR PARENTS

Helping you and your child find opportunities to discuss mathematics

Why is it important for my child to learn mathematics?

"Learning mathematics is about looking at what you thought you understood and seeing that there is deeper mystery there than you realised."
- Dan Finkel (Teacher trainer)

The mathematical skills of problem solving and reasoning contribute to having a sound understanding of the world around us. They build **confidence**, **resilience** and **curiosity**. Critical thinking skills and an ability to **communicate** our reasoning form the basic needs of many workplaces, but also help foster positive well-being and **independence**.

In our everyday lives, thinking mathematically enables us to:

- Pose and solve problems, assess information/risks and make sound decisions
- Explain how we solved a problem and why we made a particular decision
- Be confident and creative in everyday situations that with technology are changing rapidly.

Mathematics at home is not about completing lots of mathematical exercises. It is an opportunity to work practically indifferent situations where your child can see mathematics as more than just numbers on a page. It does not need expensive equipment, everyday objects are great.

- Cooking/ decorating/ gardening
- Playing games and tidying up
- Planning trips (simply deciding when to leave the house to get somewhere on time)

The aims of this guide

"Whether it is mathematics, teaching, parenting, or other areas of your life, it is really important to believe in yourself, to believe you can do anything. Those beliefs change everything."
– Jo Boaler (Professor of Mathematics Education at Stanford University)

The guide is to help you and your child be *creative* in seeing and exploring your surroundings in a mathematical way. To highlight that mathematics is more than calculating with numbers, that mathematics helps develop thinking and decision-making and supports the ability to solve problems. In going on this journey, all you need is some *curiosity* and a little self-belief. Along the way you and your child will probably lose and gain confidence repeatedly in your ability to see and discuss mathematics, but this is natural as you develop resilience and understanding as well as joy at seeing your child do it for themselves.

Keep going, keep questioning, *keep thinking*. With an open mind, there is no limit to what you can discover.

It is not the aim of this guide to teach particular strategies of calculation. If you meet a calculation you cannot solve, don't be embarrassed - get help from friends, school or social media and have a mathematical conversation. It is OK not to know. It is more important to want to find out.

Mathematics is a language and needs to be learnt through discussion. There are links to glossaries and bi-lingual dictionaries later in this resource to support talking mathematically with your child. (Appendix 3)

Learning is most effective for a child when it is in partnership with the significant individuals in their lives. The activities mentioned in this guide are not intended to replace school life, rather supplement it. Time is a key factor in learning and the more we can bring mathematics into everyday activities, from a very young age. The more time your child is learning without realising it. Little and often helps consolidate learning over time.

Everyone can do mathematics, some children just need **more time** doing it.

This guide will encourage you to look for opportunities to **spot patterns, visualise, classify, count, measure, think, reason, question**. This creates extra time for mathematics and over time promotes a deeper understanding of our surroundings. As your child gets older, they develop these skills further as the quantities get larger and the patterns more complex.

For the purposes of this guide, the word "parent" is meant to include guardians, caregivers, and other family members and friends who can help children learn mathematics.

The word "child" is intended to include children of all ages and can be collectively replaced by "children" as mathematics is a group activity.

The questions throughout the booklet are as much for you as for your child.

Do have a positive attitude to mathematics, despite your own view of the subject. It will help your child be more successful.

This includes our body language and throw away comments such as **"I was never any good at maths".** Try not to share your own anxieties.

Don't worry about knowing the right answer, but know good questions. They will develop your child's reasoning. In trying to convince you what they are doing, your child will often solve a problem themselves. They tend to realise their own mistakes and it takes the pressure off you needing to know everything. Encourage them to explain the meaning of their answers e.g. responding to an answer of, "the cake weighs 2000kg", with "Oh that sounds interesting/odd……….what would it look like?"

Don't teach your child methods that they cannot explain. Methods/tricks for getting answers can hide gaps in understanding that can take a long time to resolve. Mathematics is much more than memorising facts and procedures.

Do approximate first and use technology to calculate such as calculators and computers to support and check understanding, where appropriate

Do accept your child not knowing, provided there is a desire to know. "I don't know, but I can try to…." Don't accept "I don't get it"; ask your child for a question they would ask to help their understanding.

Don't expect your child to do maths quickly. Fluency comes over time, at and different times for different children. When we are ready, we revise our methods and make them more efficient. Unnaturally rushing this process causes stress and risks creating gaps in understanding. Therefore there are no development timelines in this guide.

Do try and answer the same question in more than one way. There is always more than one way of getting to an answer and sometimes more than one answer. Answering one question in multiple ways develops a deeper understanding of effective strategies and when to use them.

BE CREATIVE & HAVE FUN

Pattern spotting is happening all the time

"Although the world looks messy and chaotic, if you translate it into the world of numbers and shapes, patterns emerge and you start to understand why things are the way they are."
– Marcus du Sautoy (Professor of Mathematics at Oxford University)

Pattern spotting is an integral process we do subconsciously all day, using all our senses in order to make decisions.

Encourage your child to spot patterns. Talking about these patterns is very mathematical and an important part of your child's development. Asking the question **"What can you see?"** illuminates the patterns and connections your child makes.

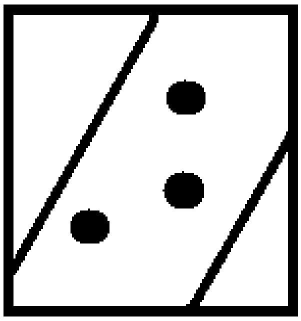
Remember that what they describe are their observations; their view of the world.

(Unless they ask) try not to superimpose our own patterns on them; our view of the world.

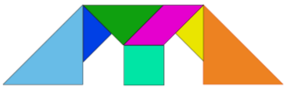
What can you see?

A leaf or a triangle?

A square, two parallel lines and three dots or…. a giraffe passing a window?



The edge patterns along each page are based on the artwork of Piet Mondrian. There is an excellent puzzle/challenge based on these rectangular shapes set by Gordon Hamilton of MATHPICKLE.COM (http://mathpickle.com/project/mondrian-art-puzzles/), a great website of graded resources including unsolved problems that are still accessible to younger children.

There are TANGRAMS  at the foot of every page.

These seven shapes can be arranged (without overlap) into many different patterns. They are reputed to have been invented in China one thousand years ago. Playing with them is a great way to develop spatial awareness, proven to improve with practice.

There is a set to cut out at the back of this booklet.

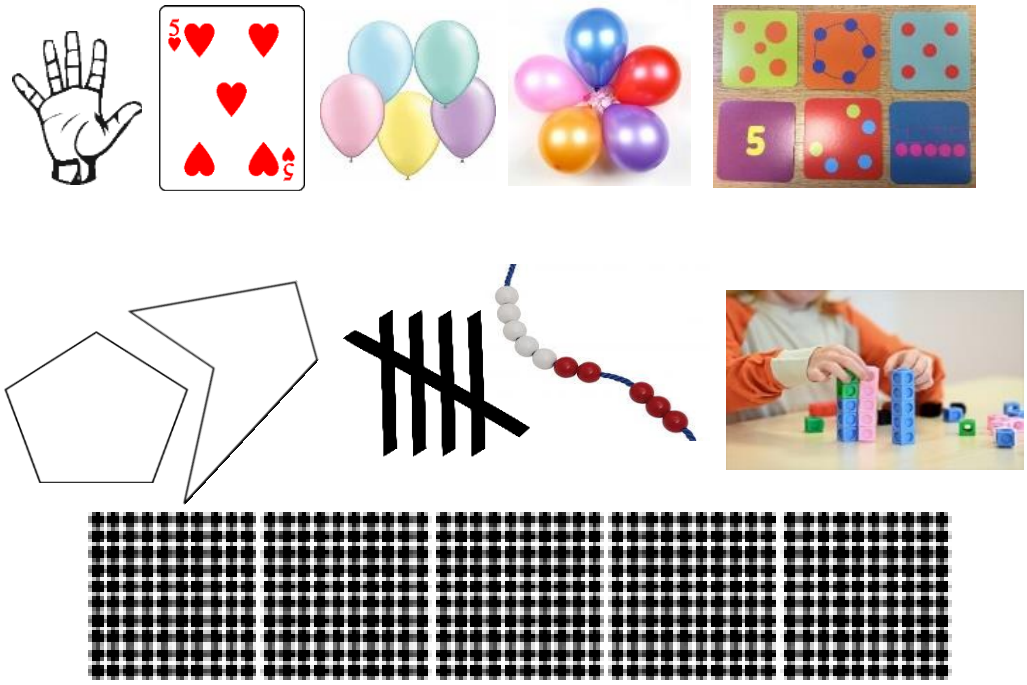
Visualising is key

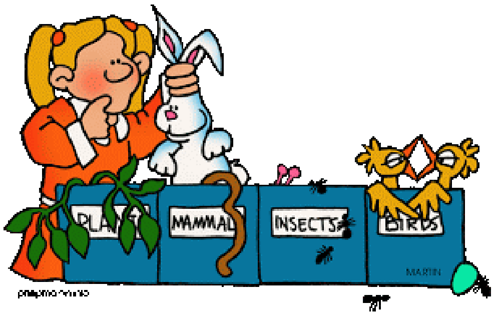
When we think of maths, we use 5 brain pathways and 2 of them are visual. It helps our brains to think of maths visually, not just in numbers. You can draw a picture of ANY mathematical idea!
- YOUCUBED (an organisation inspiring, educating and empowering teachers of mathematics)

Create (mental) images to help form mathematical concepts. Either through pictures or through stories it is important for your child to have strong images to support deep understanding. In fact, the inability to form an image can often point to a misconception or gap in understanding (even if a correct answer can be given) and may cause difficulties later.

Let them, where appropriate, touch, point to and move objects. They are exploring and they are, learning. In fact, make sure they see and play with different forms of the same number or shape. Important with fractions too.
Practice finger representations. Label their fingers with colours then numbers. Touch one of your child's fingers and without them seeing, get them to tell you the colour/number. Get them to touch something with the same colour or number.   We must do mathematics with our hands.

How do you see five?





Classifying is mathematics

"You may ask what is the use of classification, arrangement, and systemization.
I answer you: order and simplification are the first steps toward the mastery of a subject"
– Thomas Mann (German novelist and Nobel Prize winner)

Put objects into groups/boxes (such as when tidying up). This is great practice of thinking mathematically and something we do naturally every day. It is important that your child experiences sorting with physical objects and with different criteria such as shape, size, colour and texture.

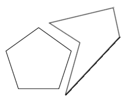
What groups of fruit do you see?

If someone else groups them can you guess the criteria they used to classify each group?

E.g. is red or has leaves

Have different forms of the same type of object when trying to classify objects (different colours/sizes). This helps to minimise misconceptions and allow precise definitions of them.

If we have only ever seen green apples, we might say any red apple  cannot be an apple.

Both of these shapes  are pentagons, but the one on the left is much more common and recognisable.

Deciding which object is different from the rest in a group is a great activity, especially if any of the objects could be the "odd one". Hearing the reasons for their choice can be very revealing about their level of understanding.

Counting is measuring

"Arithmetic is being able to count up to twenty without taking off your shoes."
– Mickey Mouse

We love to know how big groups are so we count them.

Count everything, real things. Pieces of potato on a plate, socks, pebbles, shells, fingers……. Count when tidying up to check nothing is missing. Play and talk numbers throughout the day. Count backwards as well as forwards. Sometimes start from zero.

The –*teen* numbers, such as thirteen, are difficult in English, use phrases such as 10 and 3, or one ten and three, while your child learns the new names. After all we say one hundred and three.

Value mistakes and choose when to challenge them. Twenty-eight, twenty-nine, twenty-ten is logical (as in France soixante-dix, sixty-ten for seventy). Your child is not wrong, they are learning and trying to follow a pattern, which is great. Continuing the pattern allows your child us to see that twenty-twenty-one is less logical, so we can suggest we should learn a new name as it's always a new name after something-9.

Suggest an answer/approximation before counting. Making a suggestion like "it looks like we have 8 apples here" **before** you start counting helps your child focus on the whole amount.

Count into a container. This can help your child consolidate the idea of 1 to 1 correspondence (one number name per object). It helps to initially count more accurately than pointing with a finger, if they can physically move the object.

Start small. Initially count up to five, (see the appendix 1: the principles of counting). Then count to ten in the same way and subsequently beyond. The key here is obtaining a deep sense of a number rather than rushing to tackle larger numbers. Here, bigger is not better. Remember zero.

How many hearts do you see?

1, 2, 3 …. 13, 14 (by counting) or 4 and 2 groups of 5, (written 4 + 2 x 5)

or 4 groups of 3 and 2 or even 3 groups of 5 take 1 or…………

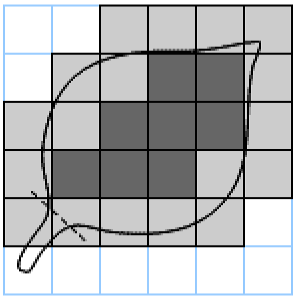
Get larger. As your child gets older, estimate/count the size of larger groups. They should no longer be counting larger quantities in ones, but unitising (seeing whole groups) and using other strategies to count (such as for adding 9, add 10 and subtract 1).

If this is not happening, they need to go back and practise with smaller numbers again. Counting on fingers is not wrong, if it supports more complex strategies than just counting in ones.

…..and measurement is counting. Practising measuring different quantities either informally or more formally with standard units makes for great mathematics.

Informal

Formal



1 square = 1 cm^2

Your child will need to measure areas, which is often about counting in squares. Formulas for finding areas are just efficient counting methods often involving multiplication. The key is still to understand what you are counting.

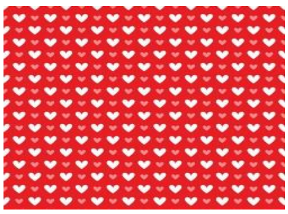
As they get older, it helps children to realise that the skills they learn such as arithmetic and measurement are essentially ways of counting. They help to count quantities they cannot touch, e.g. angles which count the amount of rotation between two directions.

Don't forget to estimate before counting. Estimates engage us to want to know the actual answer, it can be fun to see how close we are. An estimate cannot be wrong. They can be refined with more information. Often just being able to improve a guess shows a high level of reasoning. **Some of the most famous mathematical problems only have estimates as solutions** (e.g. the shortest distance between two points on a large and complex map). Making reasonable estimates develops really useful strategies for successful mathematical thinking, e.g. how do you guess how many sweets there are in the jar?

Compare the sizes of different groups. It is important that your child compare the relative sizes of two groups, being able to say which has more or less or the same.

In which picture are there more hearts?

Do we need to count?



Thinking and reasoning

> "Reserve your right to think, for even to think wrongly is better than not to think at all."
> – Hypatia (First recorded female mathematician around 400A.D.)

Mathematics is about forming convincing points of view. You need to be able to

- convince yourself
- convince a friend or family member
- convince anyone (e.g. your teacher)

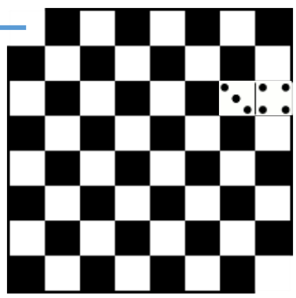
Encourage your child to initially convince themselves what they are saying is true. This is not necessarily on a mathematics question, but it could be about why they want something or want to do something. Also, explain your reasoning to them on decisions that you have made. "We are going to the park because….." This modelling helps them see how to discuss and form a convincing point of view.

Do not worry if, after your child has explained their reasoning, you do not know whether they are right or wrong.

Respond to your child's work with "You have convinced me, but we could both be wrong!" or "I'm not convinced, how else could you explain it to me, **can you do it a different way**?" This is sometimes a good idea even if you are convinced.

The very fact that your child is being asked to reason (whether correctly or incorrectly) aids their development. If incorrect, then when they correct themselves learning is at a deeper level.

Being able to effectively communicate our ideas and convince others by explaining our thinking clearly and making it transparent is a key skill in the 21st century. With the onset of computers, it is more valuable than being able to calculate quickly.



This chessboard has missing opposite corners.

Can you cover the remaining 62 squares, with 31 dominoes (like the one shown in the picture)?

Seems simple enough. However, there are 32 black and 30 white squares to cover. As each domino covers 1 black square and 1 white square, 31 dominoes cover 31 black and 31 white squares. Hence this board is impossible to cover. CONVINCED?

Questioning is just to try to learn

"I asked questions; I wanted to know why.
They got used to me asking questions and being the only woman there."
– Katherine Johnson (NASA Mathematician on the Apollo programme)

Curiosity, resilience, independence and confidence are essential components that your child needs to build fluency and subsequently retain the mathematics that they learn. It is OK not to know all the answers as long as we help them develop strategies to find them out.

Previously, on page 8, if you asked "what about the chocolates under the lid?" Then well done. You are thinking mathematically. Did you decide to include them or not?

Do activities together and discuss them. You will, in fact, be supporting their mathematics as well as helping them make decisions and judge risks that will support them throughout their lives.

If they are playing, resist asking questions that do not relate to the context of your child's play.

E.g. don't ask "how many blocks are there?", if it takes away from the activity your child is engaged in.

Ask questions based in that context

e.g. "How many people are in the pink tower?" "How many people might be in the blue tower, more or less than the pink tower?", "Why is that?"

Being questioned by your child is not easy, but both the questions of your child and your questioning of them is all part of their learning process.

Remember, when we ask a child WHY (do that!), we are trying to see what our child understands, i.e. we want to learn what they know or what they are thinking. When your child asks WHY- they are just wanting to learn.

What questions come to mind?

Knowing beyond reasonable doubt

"Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. You go into the first room and it is dark, completely dark. You stumble around, bumping into the furniture. Gradually, you learn where each piece of furniture is. Finally, after six months or so, you find the light switch and turn it on. Suddenly, it is all illuminated and you can see exactly where you were. Then you enter the next dark room…"
- Andrew Wiles (professor of mathematics at Princeton university, finally solved a problem that had taken 350 years)

There is a lot of joy in discovering something new and sharing it with others. We do not like to see our children struggle, and we want them to know the answer, so often we help them too quickly and can actually take away their satisfaction of discovering for themselves. **The answer may not be a new idea, but it will be new to them**. The joy of success gives confidence and curiosity to move on. Just seeing the correct answer can leave us feeling inferior for not finding it ourselves.

Discuss struggles; it makes us vulnerable, yet paves the way to develop our understanding. This is more than learning from mistakes. A mistake is when we do not do what we intend to do. Often what we label as a mistake isn't, as we did what we planned to do. We just had the wrong plan. We learn far more when we realise what we planned to do does not work.

There is rarely any such thing as a silly mistake. The mistake will have been caused by something, i.e. trying to complete the task too quickly or having a fundamental misconception.

8 + 6 = 2. Is this correct?

Have I made a mistake? Or is it a mistake in terms of what you are thinking?

Does a picture help?

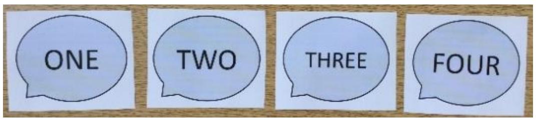
Try the same problem in a different way can help to clarify why a mistake was made and deepens understanding. Knowing how to make something work is just the start. We can learn more from solving one problem in five different ways than solving five different problems once. In time, we learn how to do it more efficiently. However, **time** is the key. We need to work through one method and get frustrated to appreciate a different method. Please hold back from teaching what you see is a better method to your child if they cannot understand why it works, which is essential if they are to connect their learning to a related problem.

Doubt must not be seen as a bad thing, it makes us open to new ideas, to fill the gaps in our thinking. It is harder to learn something new when we believe we know it already. "I'm not sure of the answer to that" is a common response to questions by people who study mathematics at the highest levels. It is seen positively because they want to find out. Not knowing does not make us weak at mathematics. It is not being open to new ideas that makes us weaker.

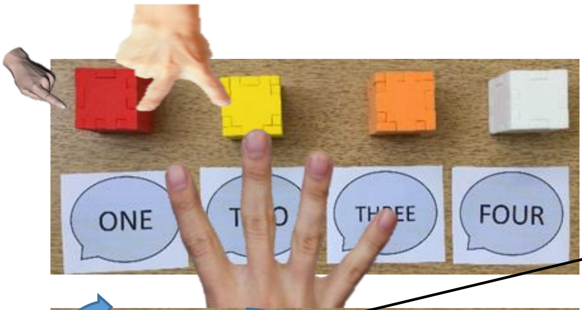
Be positive with your child if they say "I don't know" and help them *be curious.*

Appendix 1

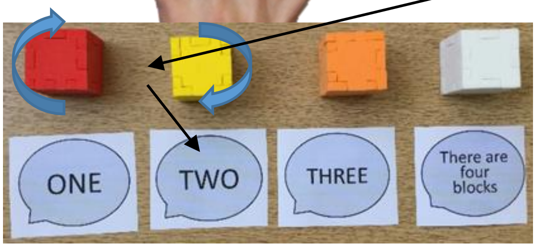
Principles of Counting



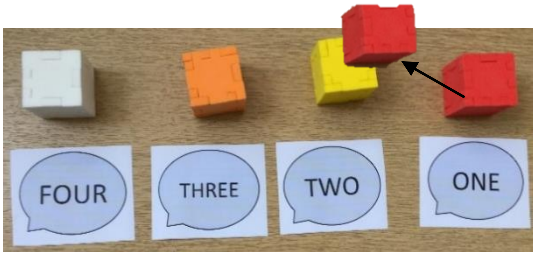
- Always say the numbers in the same stable order 1, 2, 3… (ordinality). Play clapping games, sing songs.



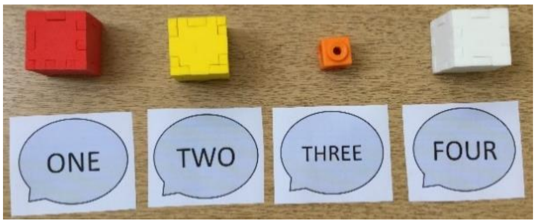
One number word for one object (one-to-one principle). Touch with fingers, moving the objects into a group (container), if necessary. Circle round with a finger each mini group as you count.



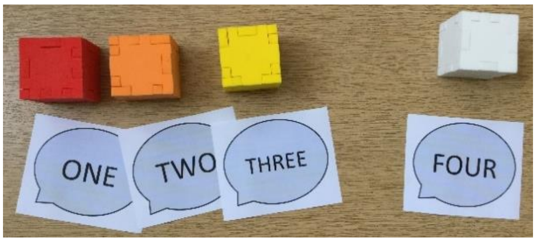
- The last number counted indicates the amount (cardinality). Repeat with the question after counting. So how many? The younger the child the more likely they are to recount. Keep asking – eventually you will get an answer without a recount.



- The order that objects are counted in is irrelevant (start counting from the end, in the middle, right to left, up and down). Again make mini groups as you count each object in the whole group to emphasise the word name.

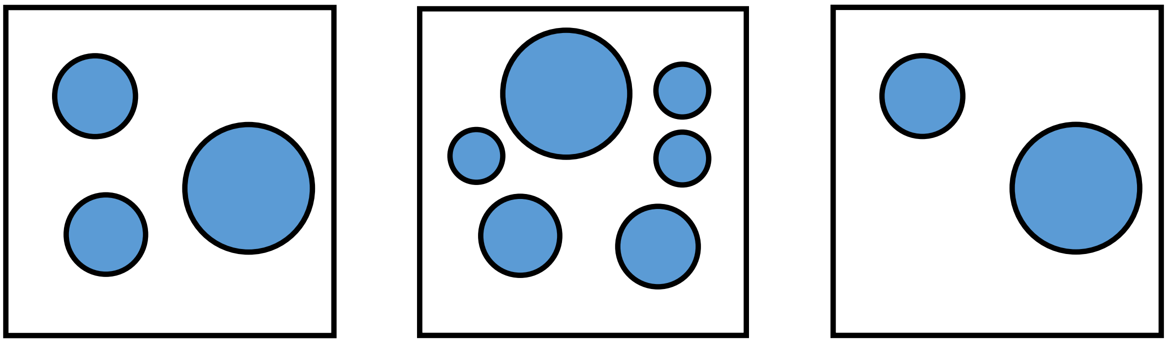


- The adjective (e.g. size or colour) is not important. It is the noun, what we are counting, that matters e.g. 1 block, 2 blocks. The noun can be non-physical like sounds.



- The position of the objects is not important. Learning different representations of a number is important.

These principles are intrinsic in being able to count. Difficulties can sometimes be highlighted with simple dot patterns.

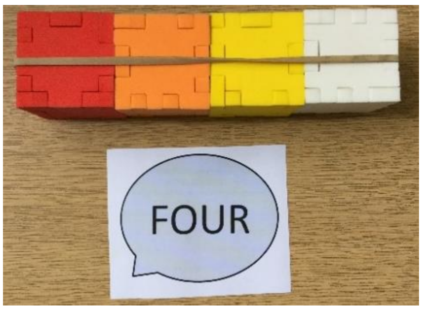


While questions such as

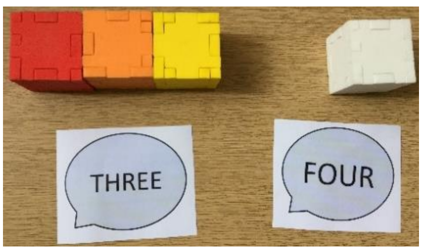
- How many dots?
- Which square has more dots?
- Which two squares make five dots?
- Which square can be represented by the digit 6?

may appear easy, they are fundamental in our ability to work mathematically. If your child struggles to answer such questions, it can point to long term difficulties in number and needs to be highlighted early on.

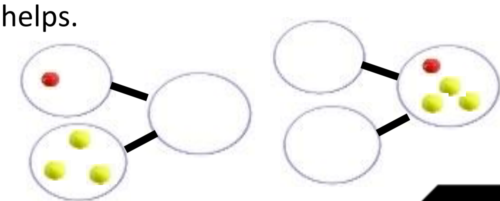
The patterns can be traditional such as on dice or playing cards or more randomly arranged. The larger the number of dots the harder it is to know the quantity without some counting strategy. If your child struggles to answer, try fewer dots.

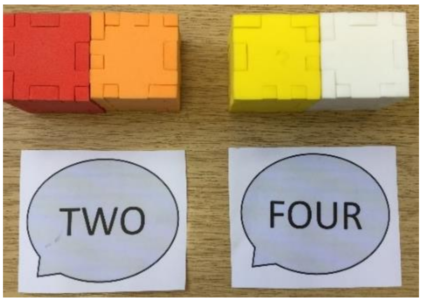


- Subitising groups of objects. Seeing a group as a whole without the need to count because of its structure. Elastic bands (or putting on plates) help children to see a number of objects as one group of say 4. In the middle dot pattern above, do you group the dots for easier counting?

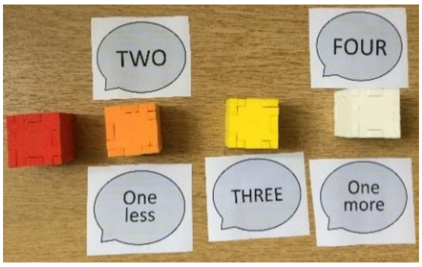


- Realising numbers are made up of other numbers (inclusion). Physically moving the items from the parts to the whole and from the whole to the parts, helps.





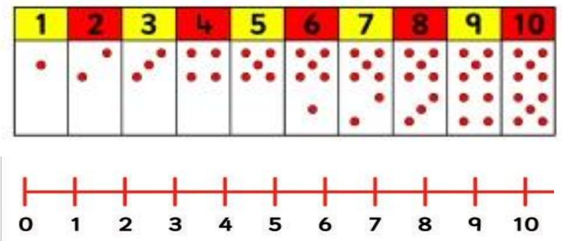
- Unitising (counting in twos, fives, tens etc.)
While unitising is important, learning multiplication table facts should never be attempted as a rote memorisation activity.
It is not a speed exercise either.

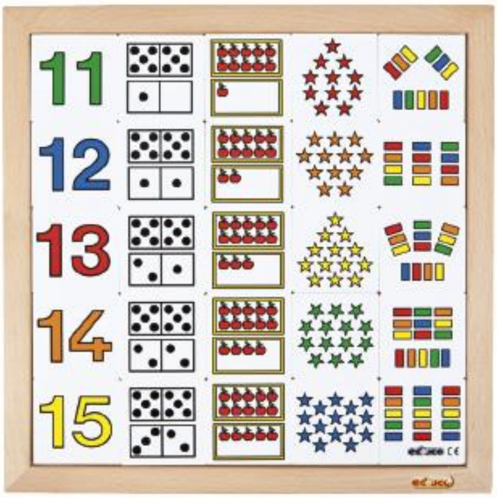


- Understanding the relative position of numbers to each other. Initially number tracks, that have no spaces between numbers, reinforces this more than a number line. Use the language, one more and one less, (not add 1 or 1 after, which comes later, as does two more/two less). Games with a scoring track can help here.

Please make sure your child is secure with all the numbers up to 10 i.e. they know six is made up of five and one or four and two or even two threes, before counting to higher and higher numbers, for which they may know the name but not truly grasp the number. Do we really grasp what a million or billion is when we hear it used? Your child will then ready make the connection that 60 i.e. 6 tens will obviously be say 4 tens and 2 tens (40 + 20).

Which displays supports learning to count?



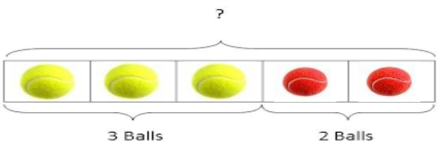


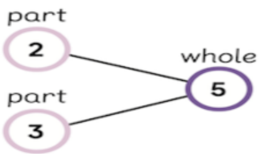
Appendix 2

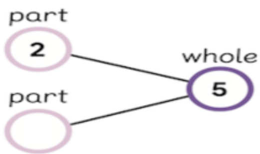
"Many who have had an opportunity of knowing any more about mathematics confuse it with arithmetic, and consider it an arid science. In reality, however, it is a science, which requires a great amount of imagination."

– Sofia Kovalevskaya (First female professor of mathematics 1880s)

Addition and Subtraction problems can be seen as two separate parts of a whole group. This is often represented by a part-whole diagram. Hiding part of the number can be helpful and fun here.







It can be useful to emphasise the noun of each part and the whole. e.g. 3 (yellow) <u>balls</u> and 2 (red) <u>balls</u> make 5 <u>balls</u>.

Then later it makes sense to replace the noun with mathematical terms e.g. 30 + 20 said as 3 tens and 2 tens make 5 tens, or 0.3 + 0.2 said as 3 tenths and 2 tenths make 5 tenths (0.5) and so on.

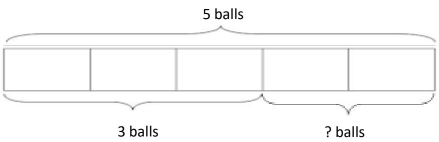
KEY STRATEGY: Know which numbers add to make 10.

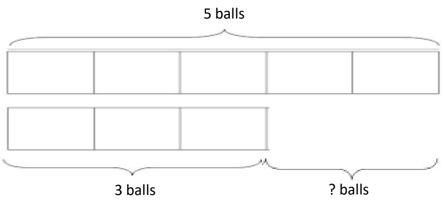
Alternatively addition and subtraction can represent changes over time. After 2 ducks swim away, 3 ducks are left. How many were there in the beginning?

Finally addition and subtraction can be used to compare two groups. The sister has 3 more sweets than her brother.

"What's the story?"

I have 5 balls. I lose 3. How many do I have left? (change over time)
I have 5 balls. My friend has 3. How many more balls do I have? (comparison)
We have 5 balls and he has 3. How many do I have? (part-whole)





Each question uses the same calculation but has a fundamentally different structure
Which diagram helps understand each question?
Looking at one calculation in multiple ways is more beneficial than looking at many calculations once.

Multiplication puts individual (items) ones into groups (e.g. threes or fours). Counting these groups involves skip counting (repeated addition) and in time recognising standard results.

It can help to relate multiplication to a visual context.

Is 4 x 3 the same as 3 x 4?

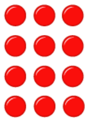
For example, 3 vases with 4 flowers in each vase has 12 flowers altogether. 4 vases with 3 flowers in each vase also has 12 flowers altogether. The total number of flowers are the same but the number of vases is different.
Therefore to learn the multiplication number facts, it is important to understand the context of a multiplication and not to just recite by rote.

A lot of time and effort is given to learning "multiplication tables". The values formed from multiplying two single digit numbers together simply become a new set of numbers names e.g. 4, 8, 12, 16, 20 ……..just like 1, 2, 3, 4, 5 … in the principles of counting.

To make learning these new patterns effective, keep it fun and……

1. *Learn to skip count* e.g. 1, 2, 3, **4**, 5, 6, 7, **8**, 9, 10, 11, **12**, then remove the intermediate numbers to leave 4, 8, 12. This is forming a new stable order of numbers names as happened when learning to count in ones.

2. *Visualise.* Draw an array of dots to represent each item



3. *Deduce new facts from known facts.* Once the size of small arrays such as 4 groups of 3 are known (subitised), new facts can be deduced more easily.
   KEY STRATEGIES
   doubling: 8 x 3 (8 groups of 3) is double 4 groups of 3 (12 + 12 = 24)
   adding: 8 x 3 (8 groups of 3) is 5 groups of 3 + 3 groups of 3 (15 + 9 = 24)
   subtraction: 8 x 3 (8 groups of 3) is 10 groups of 3 – 2 groups of 3 (30 – 6 = 24)

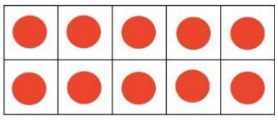
4. *Say complete sentences in context*, when practising tables,
   3 bags of 8 sweets = 24 sweets.

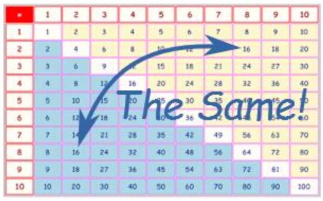
Do not focus overly on speed or memorisation, as these can lead to anxiety if your child struggles.

The commutativity of addition and multiplication means that we can calculate in either order, e.g.

3 + 4 = 4 + 3 and 3 x 4 = 4 x 3. Evidence suggest that we organise (remember) the number facts, preferring when the larger number comes first.

Learning that 2 groups of 5 make 10, allows us to reorganise our thinking to give us that 5 groups of 2 must also be 10.





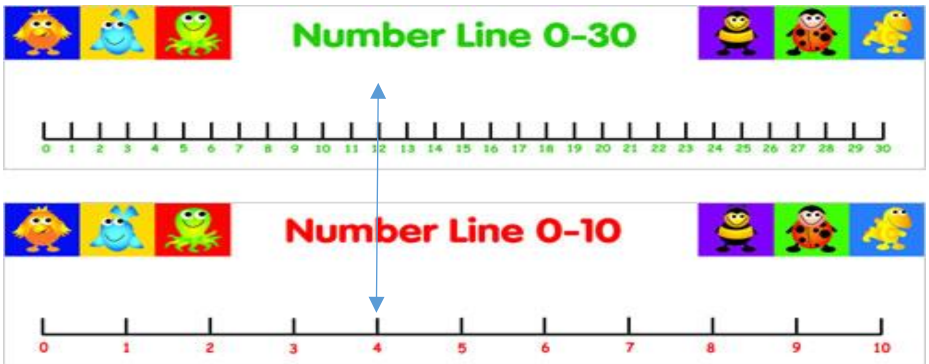
So learn 2 x 2 to 2 x 9, then 3 x 3 to 3 x 9 and so on, as 3 x 2 can be thought of as 2 x 3.

Only going as far as 5 x 5 initially also cuts down the amount to focus on.

It can make these facts more secure in our memory.

Please remember to always focus on visualising with a noun (e.g. the flowers) rather than just memorising numbers, which will come with practice. This supports later multiplications e.g. 0.3 x 4 = 3 tenths x 4 = 12 tenths = 10 tenths and 2 tenths = 1 whole and 2 tenths = 1.2

Later multiplication is linked to the scaling of the number line. Instead of multiplication being seen as relatedly adding the same number to the total, it is seen a stretching and shrinking.



The spaces on the red number line are three times larger than on the green line.

This move to multiplicative reasoning rather than additive step counting is very important in understanding fractions, ratios and proportions.

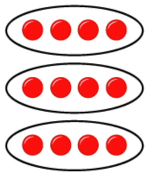
Comparing the amount of water in each glass.

The difference in the amount of water between the glasses on the left is the same as between the glasses on the right. However, one of the glasses on the right has twice as much as the other, but this is not the case on the left.

Division can be thought of in two ways. Grouping and sharing. It is complementary with multiplication.

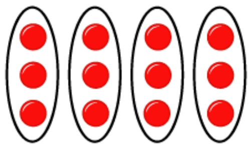
Grouping – we know how many in each group and we want to know how many groups.

Read/say 12 ÷ 4 as 12 grouped into groups of 4. Therefore, 3 groups.



Sharing – we know how many groups are needed, but do not know how many in each group.

Read/say 12 ÷ 4 as 12 shared into 4 groups. Therefore, 3 in each group.

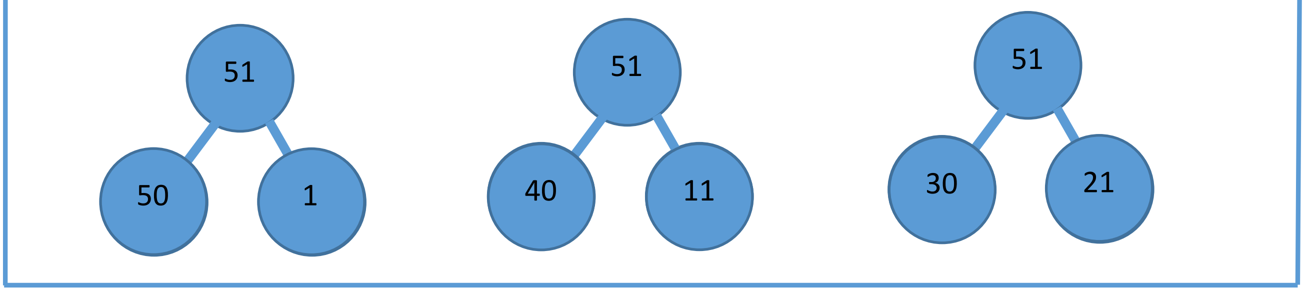


They are interchangeable. Using this language of grouping and sharing with your child will support their conceptual understanding and allow them to make effective mental pictures when calculating.

Partitioning

Encourage your child to look to partition multi-digit numbers (splitting a number in part) in different ways. This helps to calculate more efficiently.

Which partition helps calculating 51 – 29? Which helps 51 ÷ 3?



Renaming

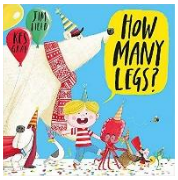
Renaming a number means to change a number's noun(s). E.g. 1900 one thousand nine hundred is the same as nineteen hundreds. This, like partitioning, can help to calculate more efficiently.

400 = 4 hundreds. Renamed becomes 40 tens. So 400 ÷ 5 = 40 tens ÷ 5 = 8 tens = 80

$\frac{1}{2}$ ÷ 3 One half is renamed as 3 sixths. 3 sixths ÷ 3 = 1 sixth = $\frac{1}{6}$ (we say one sixth not one over six)

Appendix 3 Useful (but not essential) resources

TV/Books/Multimedia:



Numberblocks –cbeebies
Storybooks (bedtimes can be mathematical too)
Maths for Mums and Dads – Eastaway and Askew
Mathpickle.com
Brainingcamp apps

Language dictionaries and glossaries: https://acceal.org.uk/maths/

≤	smaller than or equal to	*menor ou igual a*
%	per cent	*por cento*
:	ratio	*razão*

Games:

- Any game with standard playing cards
- Any game with dice
- Carcasonne
- Dominoes
- Guess Who
- Pass the Pigs
- Patchwork
- Snakes and Ladders
- Sushi Go Party

Puzzles:

- Any SmartGames Puzzles
- Any ThinkFun Puzzles
- Pentominoes
- Soma Cube
- Tangrams
- Tantrix

Appendix 4 Statements and questions to aid mathematical discussion

Sentence starters for children

The first thing I did was ....
I already knew ....so ....
I noticed that ....
Once I found out .... I could then ....
It didn't work when I .... so I ....
The part I found most difficult was .... because ....
The part I found easiest was .... because ....
It could be .... because ....
It couldn't be .... because ....

1. *Starter questions*

These take the form of open-ended questions which focuses your child's thinking in a general direction and gives them a starting point.

Examples:
How could you sort these.......?
How many ways can you find to .... ?
What happens when we .... ?
What can be made from....?
How many different .... can be found?

2. *Questions to stimulate mathematical thinking*

These questions can assist your child to focus on particular strategies and help them to see patterns and relationships. This aids the formation of a strong conceptual network. The questions can serve as a prompt when your child becomes 'stuck'. (It is often tempting to turn these questions into instructions, which is far less likely to stimulate their thinking)

Examples:
What is the same?
What is different?
Can you group these .... in some way?
Can you see a pattern?
How can this pattern help you find an answer?
What do think comes next? Why?
Is there a way to record what you've found that might help us see more patterns?
What would happen if....?

3. *Assessment questions*

Questions such as these ask children to explain what they are doing or how they arrived at a solution. They allow you to see how your child is thinking, what they understand and what level they are operating at. Obviously they are best asked after your child has had time to make progress with the problem, to record some findings and perhaps achieved at least one solution.

Examples:
What have you discovered?
How did you find that out?
Why do you think that?
What made you decide to do it that way?

4. *Final discussion questions*

These questions draw thoughts together and prompt sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking processes. It provides further opportunity for reflection and realisation of mathematical ideas and relationships. It encourages children to evaluate their work.

Examples:
Is there a different solution?
Why/why not?
Have we found all the possibilities?
How do we know?
Have you thought of another way this could be done?
Do you think we have found the best solution?

What would you change to create a new problem?

It is also very beneficial to encourage your child to come up with their own questions and explore them

(based from NRICH – Using Questioning to Stimulate Mathematical Thinking)

Appendix 5 – Suggested ideas

Early Years Foundation Stage/KS1

Maths for a purpose and audience	Developing a love of maths
- Help to follow a recipe, using scales, cups and spoons, talking about measure. - Paying for items in shops using notes and coins. - Looking for shapes and patterns in the environment, use mathematical language to describe them, curved, straight, flat, solid… - Start to understand the concept of time, talking about routine, discussing different times of the day. - Water play filling containers and introducing vocabulary such as full and empty.	- Portray confidence, see and explore, encourage listen and engage your child's mathematical talk e.g. singing counting rhymes and mathematical stories. - Showing and sharing your enjoyment of mathematics by playing games such as, dominoes, snap and sharing stories. - Make time to explore and repeat activities. - Looking at number plates, door numbers, and road signs when walking – mathematics is all around us. Sharing a love of size (counting) and pattern spotting, helps not just for when your child gets to school, but also in the years beyond.

Building blocks for maths	Developing fluency
- Find lots of opportunities to talk to your child about Maths in everyday contexts e.g. counting steps and matching pairs of socks. - Provide lots of opportunities for your child to develop their mark making skills by providing a variety of writing implements e.g. sticks in the sand, paint, chalk. - A child's daily life provides practical opportunities to learn about number, shape and space e.g. tidying up put similar objects together, steering a pram, how many knives and forks needed when setting the table. - "Show me 5 pieces of fruit that are not all the same". - Mention number facts, e.g. 5 is made up of 2 and 3.	- Ask your child questions and help to expand their answers e.g. 'I wonder why…' - Support your child by practising skills in a fun way counting forwards and backwards aloud - Repeating favourite maths activities to allows your child to become familiar and to consolidate previous learning. - Practice finger representations. Label the fingers with colours then numbers. Touch one of your child's fingers and without them seeing, get them to tell you which one. Get them to touch something with the same colour or number.

Early Years Foundation Stage/ KS1

Taking a risk	Independence
- Encourage your child to always have a go. - Encourage your child to ask questions. - Show your child that it is ok to make a mistake. - Praise effort and all attempts even if they are not quite right. - Make predictions and see what happens. If children hear I can't do maths from their parents, or friends then they begin to believe that it isn't important or that it's ok to not try when doing Maths. Encourage an enjoyment and confidence of learning about the world around us mathematically.	- Ensure that a range of mathematical opportunities are available and easily accessible in the home e.g. pine cones for counting, blocks for building, stories with numbers in. - Allow your child **time** to have a go before supporting. - Model mathematical talk aloud when doing tasks as an adult e.g. I used the triangle because...

Checking answers	Discussing and visualising
- Show your child a mistake you have made and how to put it right. - Model making a mistake and correcting yourself e.g. saying,"1, 2, 3, 5 no 1, 2, 3, 4." - If your child mispronounces a word e.g. name of a shape, model the correct language without drawing attention to your child's mistake.	- Discuss new ideas and experiences from nursery and school, visits, books and TV programmes. - During physical activities, encourage mathematical learning by counting when kicking balls, climbing and running. - Develop mathematical talk during imaginative activities such as counting coins when playing shops. - Growing plants and talking about changes. - "Each piece of fruit costs £1. How could you spend £5?"

KS1/KS2

Mathematics for a purpose	Developing a love of mathematics
- Include your child in everyday activities that involve mathematics – making purchases, measuring ingredients, scale a recipe up or down to feed the right amount of people, measuring and calculating the area of a room, and use a bus timetable to work out how long a journey between two places should take. Go on the journey. Do you arrive earlier/later than expected? By how much? - Play games and do puzzles with your child that involve mathematics. Such activities may focus on direction or time, logic, reasoning, sorting, classifying, and/or estimating.	You can help to develop your child's mathematics by: - Being positive and talking about the ways you use maths every day. - Encouraging them to be persistent when a problem seems difficult—ask them how you can work together to think of a solution and praise their effort. - Acting as a good role model by responding to and engaging with mathematics positively - Play games that require thinking and strategy. Make up games when going places. Have fun.

Building blocks for mathematics	Developing fluency
- Talk about/research topics of interest with your child. - Encourage your child to discuss strategies and find efficient methods. - Actively encourage your child to draw pictures to represent their ideas. Talk about concepts or vocabulary that may be difficult to understand.	- Ask your child questions and help to expand their answers e.g. 'I wonder why…' - Support your child by practising skills in a fun way, check they can increasingly count using mental strategies. - Repeating favourite maths activities allows your child to become familiar and to consolidate previous learning. - Help your child to learn number bonds and times tables, through reasoning and visualising— Play games.

Taking a risk	Independence
- Encourage your child to always have a go and if necessary be prepared to have a go yourself. - If you get it wrong, it is not a mistake but an opportunity to adapt your thinking - Encourage your child to ask questions. - Show your child that it is ok not to know the answer and to describe what they have tried to do to find a solution to a problem. - Praise effort and all attempts even if they are not quite right. - Make predictions and see what happens.	- Talk through your child's homework with them when they have finished. - Provide support materials—objects, ruler, calculator, number grids. - Encourage your child to investigate problems and how they might find solutions in **more than one way.** We can learn more from solving one problem in five different ways than solving five different problems once.

Checking answers	Discussing and visualising
- Encourage your child to check their answers and consider if answers are reasonable in the context of the question. - If your child describes their working in a way, you do not understand or think "I know what you mean, but it isn't clear", ask them to find a different explanation. - Have scrap paper available for working out.	- Point out mathematical shapes or arrangements when they occur during the day. - Spot patterns and wonder why they occur. - Ask, "What do you see?" and "how do you know?" - Do not rush to say if an answer is right or wrong. Get them to convince you why it is right. - Encourage using mathematical language and linked sentences including "I think….because…."

Appendix 6 - Tangram